Assignment 4: QFT

Let’s begin with how we look at 2-qubits.

For completeness, let's show all conditions using these basis. (This is the computational basis i.e. Z)

Then we have

, , ,

We can start with

Then if we try a to look at the following:

What does this mean?

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We look at the next Hadamard

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We take the first two gates with the identity

We need to solve for our initial condition

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Did you notice something? When we look at a circuit, we see H than X, but when we do the math, we have the X matrix on the right and H matrix on the left. We execute matrix for quantum from right to left, even though we see things from left to right. Something to remember.

How we can go through similar steps and derive the 2 Qubit QFT based on Aradhita’s lecture.

MATLAB exercise.

bloch0 = [1;0];

bloch1 = [0;1];

% 2-qubit QFT

% Solved equation |x1 x2>

% |x1> --H--R2-----|-- 1/sqrt(2) \* (|0> + exp (2\*i\*pi\* (x1/2+x2/4))|1>)

% | |

% |x2> -----x---H--|-- 1/sqrt(2) \* (|0> + exp (2\*i\*pi\* (x2/2))|1>)

% |00>

a1=1/sqrt(2)\*(bloch0+exp(2\*i\*pi\*(0/2+0/4))\*bloch1);

a2=1/sqrt(2)\*(bloch0+exp(2\*i\*pi\*(0/2))\*bloch1);

% |01>

a3=1/sqrt(2)\*(bloch0+exp(2\*pi\*i\*(0/2+1/4))\*bloch1);

a4=1/sqrt(2)\*(bloch0+exp(2\*pi\*i\*1/2)\*bloch1);

% |10>

a5=1/sqrt(2)\*(bloch0+exp(2\*pi\*i\*(1/2+0/4))\*bloch1);

a6=1/sqrt(2)\*(bloch0+exp(2\*i\*pi\*(0/2))\*bloch1);

% |11>

a7=1/sqrt(2)\*(bloch0+exp(2\*pi\*i\*(1/2 + 1/4))\*bloch1);

a8=1/sqrt(2)\*(bloch0+exp(2\*pi\*i\*1/2)\*bloch1);

my2QFT = [Tensor(a2,a1),Tensor(a4,a3),Tensor(a6,a5),Tensor(a8,a7)];

FourierMatrix(4) % Check the answer

% Identity

ident = Pauli(0);

% H gate

myHgate = FourierMatrix(2); % 1/sqrt(2) [ 1 1 ; 1 -1]

% X

xGate = full(Pauli(1));

% Y

yGate = full(Pauli(2));

% Z

zGate = full(Pauli(3));

% S

sGate = [1,0;0,i];

% swap

swapGate = [1 0 0 0;0 0 1 0;0 1 0 0;0 0 0 1];

% Solved with circuit

% --H------R2------------|--

% |I> |A> | |B> |C> | |Q>

% ---------x-------H-----|--

%|A> = U1|I>

%|B> = U2|A>

%|C> = U3|B>

%|Q> = U4|C>

U1 = Tensor(myHgate,ident);

U2 = rGate(2);

U3 = Tensor(ident,myHgate);

U4 = swapGate;

Q = U1\*U2\*U3\*U4;

Exercise 3:

% Solved with circuit

% --H------R2-----R3--------------------------------|--

% |I> |A> | |B> | | |Q>

% ---------x------|---------H-------R2--------------|--

% | |C> |D> | |E> |F> |

% ----------------x-----------------x-------H-------|--

\*\* I flipped my gates, but we can do something with Qiskit

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As you can see, we can build the matrix row by row using Qiskit for 3-qubit QFT. Each of the Output state is a row in the final QFT matrix.

We can quickly look using the built in on the results

>> FourierMatrix(8)

ans =

Columns 1 through 4

0.3536 + 0.0000i 0.3536 + 0.0000i 0.3536 + 0.0000i 0.3536 + 0.0000i

0.3536 + 0.0000i 0.2500 + 0.2500i 0.0000 + 0.3536i -0.2500 + 0.2500i

0.3536 + 0.0000i 0.0000 + 0.3536i -0.3536 + 0.0000i -0.0000 - 0.3536i

0.3536 + 0.0000i -0.2500 + 0.2500i -0.0000 - 0.3536i 0.2500 + 0.2500i

0.3536 + 0.0000i -0.3536 + 0.0000i 0.3536 - 0.0000i -0.3536 + 0.0000i

0.3536 + 0.0000i -0.2500 - 0.2500i 0.0000 + 0.3536i 0.2500 - 0.2500i

0.3536 + 0.0000i -0.0000 - 0.3536i -0.3536 + 0.0000i 0.0000 + 0.3536i

0.3536 + 0.0000i 0.2500 - 0.2500i -0.0000 - 0.3536i -0.2500 - 0.2500i

Columns 5 through 8

0.3536 + 0.0000i 0.3536 + 0.0000i 0.3536 + 0.0000i 0.3536 + 0.0000i

-0.3536 + 0.0000i -0.2500 - 0.2500i -0.0000 - 0.3536i 0.2500 - 0.2500i

0.3536 - 0.0000i 0.0000 + 0.3536i -0.3536 + 0.0000i -0.0000 - 0.3536i

-0.3536 + 0.0000i 0.2500 - 0.2500i 0.0000 + 0.3536i -0.2500 - 0.2500i

0.3536 - 0.0000i -0.3536 + 0.0000i 0.3536 - 0.0000i -0.3536 + 0.0000i

-0.3536 + 0.0000i 0.2500 + 0.2500i -0.0000 - 0.3536i -0.2500 + 0.2500i

0.3536 - 0.0000i -0.0000 - 0.3536i -0.3536 + 0.0000i 0.0000 + 0.3536i

-0.3536 + 0.0000i -0.2500 + 0.2500i 0.0000 + 0.3536i 0.2500 + 0.2500i

Does each row match, if you finish the rest?

What are we observing here?

A diagram of a block diagram

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Image taken from: https://young.physics.ucsc.edu/150/QFT-FFT.pdf

From Aradihta’s lecture, you should know that

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We can follow similar derivation and solve for the 3 qubit gates